ISSN: 2456-7744

Advanced Scientific Research 2017 2(2) 16-19

Advanced Scientific Research
journal homepage: www.advancedscientificresearch.in

Relativistic effects on the linear and nonlinear properties of electron plasma waves

M.P. Singh*

Department of Physics, R. R. P. G. College Amethi (U.P.), India

ARTICLE INFO

Article history:
Received 24 November 2017
Received in revised form 28 December 2017
Accepted 30 December 2017
Available online 31 December 2017

Keywords:
Relativistic
Plasmawaves
Hydrodynamic
Schrodinger equation
Electron

ABSTRACT

We present relativistic effects on the linear and nonlinear features of electron plasma waves using the one-dimensional quantum hydrodynamic (QHD) model for a two components electron-ion dense quantum plasma. In view of perturbation technique, a nonlinear Schrodinger equation (NLSE) describing both relativistic and quantum effects have been presented.

1. Introduction

Several workers have investigated the linear and nonlinear properties of plasma waves mainly confined to classical nonrelativistic plasma. In some plasma particle velocities become high approaching to the speed of light, so for such plasmas, it becomes necessary to consider relativistic effects. In fact, relativistic effect significantally modify the linear and nonlinear properties of plasma waves. Relativistic plasma may be formed in many practical situations e.g. in space-plasma phenomena and in various situations mainly laser-plasma interaction experiments [1-8]. In the literature, only a few studies related to relativistic effects on electron plasma waves may be found.

All the above studies on relativistic effects on plasma waves have been presented for classical plasma. But in plasmas, where the density is too high and temperature is very low, the thermal de Broglic wavelength may become comparable to the inter particle distances. In these situations, quantum effects come in the picture due to the overlapping of wave functions of the nearby particles. These quantum effects may modify the linear and nonlinear features from that found in the corresponding classical plasma. As a newly emerging field in plasma physics, quantum plasmas have received much attention. There has been an important interest in the study of linear and nonlinear features of various wave modes in quantum plasmas [9-11]. Many workers have presented quantum effects on linear and nonlinear features of ion-acoustic and dust-acoustic waves [12-16].

In this paper, we have presented the linear and nonlinear properties of electron plasma waves including weakly relativistic effect and ion motion, using the one-dimensional quantum hydrodynamic (QHD) model for two compound electron-ion dense quantum plasma. It is shown that the relativistic effects may change the linear and nonlinear properties of electron plasma waves in quantum plasma.

* Corresponding author. E-mail: drmanvendrarrpg@gmail.com

2. Some Basic Equations

Let us consider weakly relativistic plasma made of electrons and ions moving along the x-axis and the pressure law reads [17-18]

$$p_j = \frac{m_j V_{F_j}^2}{3n_{jo}^2} n_j^3 \tag{1}$$

where $j = e$ for electrons, $j = i$ for ions, m_j be the mass, $V_{F_j} = \sqrt{2K_B T_{F_j} / m_j}$ be the Fermi speed, T_{F_j} as the Fermi temperature and K_B is the Boltzmann constant and n_j as the number density with the equilibrium value n_{jo}. Bohm potential plays an important role in describing quantum hydrodynamics and Bohm defined wave function $\psi(x,t)$ as

$$\psi(x,t) = R(x,t)\exp\left[iS(x,t)/\ \right], \tag{2}$$

where R as the real amplitude and S is the real phase. Then, using Schrodinger equation he obtained equation for S which apart from the usual classical potential contains an additional potential term, known as Bohm potential [19].

Now one may obtain the set of quantum hydrodynamic (QHD) equations describing the dynamics of the electron plasma waves in the model plasma are,

$$\frac{\partial n_j}{\partial t}+\frac{\partial\left(n_j u_j\right)}{\partial x}=0, \tag{3}$$

$$\left(\frac{\partial}{\partial t}+u_j\frac{\partial}{\partial x}\right)\left(u_j\gamma_j\right)=-\frac{q_j}{n_j}\frac{\partial\varphi}{\partial x}-\frac{1}{m_j n_j}\frac{\partial p_j}{\partial x}$$

$$+\frac{h^2}{2m_j^2\gamma_j}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_j}}\frac{\partial^2}{\partial x^2}\left(\sqrt{n_j}\right)\right], \tag{4}$$

$$\frac{\partial^2\varphi}{\partial x^2}=4\pi e\left(n_e-n_i\right), \tag{5}$$

where u_j, q_j and p_j are the fluid velocity, charge and the pressure of the jth species, $q_e=-e,\ q_i=e,\ \gamma_e=\left(1-u^e\, e/c^2\right)^{1/2}$ as the relativistic factor for electrons, $\gamma_i=0$ for ions, C as the velocity of light in free space, h as the Planck's constant divided by 2π and φ as the electrostatic wave potential. Due to lighter mass of electrons, attain relativistic speed more easily than the heavier ions. For this one may consider the ion motion as non-relativistic. It is an important to note that we have taken relativistic effect only in the equation (4) for simplicity and ultra-cold plasma with weakly relativistic effect.

Let us use the normalisation as: $x\to x\omega p_e/V_{F_e}$, $t\to t\omega p_e$, $\varphi\to e\varphi/2K_B\,T_{F_e}$, $n_j\to n_j/n_0$ and $u_j\to u_j/V_{F_e}$, where $\omega_{pe}=\sqrt{4\pi n_0\,e^2/m_e}$ be the electron plasma oscillation frequency, V_{F_e} be the Fermi speed of electrons.

In view of normalisation, one obtains following set of equations for ions and electrons as,

$$\frac{\partial n_i}{\partial t}+\frac{\partial\left(n_i u_i\right)}{\partial x}=0, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{dx}\right)u_i=-\mu\frac{\partial\varphi}{\partial x}-\sigma n_i\frac{\partial n_i}{\partial x}+\frac{\mu^2H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_i}}\frac{\partial^2\sqrt{n_i}}{\partial x^2}\right], \tag{7}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e u_e\right)=0, \tag{8}$$

$$\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{dx}\right)\left(u_e\gamma_e\right)=\frac{\partial\varphi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2\gamma_e}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right], \tag{9}$$

$$\frac{\partial^2\varphi}{\partial x^2}=\left(n_e-n_i\right), \tag{10}$$

where

$H=\ \omega p_e/2K_BT_{F_e}$,

be a nondimensional quantum parameter proportional to the quantum diffraction, $\mu=m_e/m_i$ and $\sigma=T_{F_i}/T_{F_e}$. The quantum diffraction parameter H is proportional to $\omega p_e/K_BT_{F_e}$.

3. Nonlinear Schrodinger Equation (NLSE)

Let us assume that ions being heavier mass, may not respond to the high frequency components of the field quantities. They contribute only to the slowly-varying part of the field quantities generated through nonlinear interactions of high frequency waves. In view of the above considerations one may make the following Fourier expansion of the field quantities.

$$\begin{bmatrix}n_e\\u_e\\\varphi\end{bmatrix}=\begin{bmatrix}1\\u_0\\0\end{bmatrix}+\in^2\begin{bmatrix}n_{e0}\\u_{e0}\\\varphi_0\end{bmatrix}+\sum_{S=1}^{\infty}\in^S\left\{\begin{bmatrix}n_{es}\\u_{es}\\\varphi_s\end{bmatrix}.\exp\left(is\varphi\right)\right.$$

$$\left.+\begin{bmatrix}n_{es}^*\\u_{es}^*\\\varphi_s^*\end{bmatrix}.\exp\left(-is\psi\right)\right\} \tag{11}$$

$$\begin{bmatrix}n_i\\u_i\end{bmatrix}=\begin{bmatrix}1\\v_0\end{bmatrix}+\in^2\begin{bmatrix}n_{i0}\\u_{i0}\end{bmatrix}, \tag{12}$$

$$\xi=\in\left(x-cgt\right)\quad\text{and}\quad\tau=\in^2 t, \tag{13}$$

where $\in$ as a small parameter and C_g as the normalised group velocity. By substituting eqs. (11) and (12) in eqs. (6)-(10) and then equating both sides the coefficients of $\exp\left(i2\psi\right)$ and terms independent of ψ, one obtains three sets of equations which one says I, II and III. In order to solve these three sets of equations one makes the following perturbation expansion for the field quantities $n_{eo}, u_{eo}, \varphi_0, n_{es}, u_{es}, \varphi_s, n_{io}$ and u_{io} which one denotes as A:

$$A=A^{(1)}+\in A^{(2)}+\in^2 A^{(3)}+\dots\dots\dots \tag{14}$$

By solving the lowest order equations obtained from the set of equations I after substituting eq. (14), one obtains,

$$n_{e1}^{(1)}=-k^2\varphi_1^{(1)}, u_{e1}^{(1)}=-k\left(\omega-ku_0\right)\varphi_1^{(1)}, \tag{15}$$

and the normalised dispersion equation

$$\gamma_3\left(\omega-ku_0\right)^2=1+k^2+\gamma_2\frac{H^2k^4}{4}, \tag{16}$$

$$\text{where}\quad \gamma_3=1+\frac{3u_0^2}{2c^2},\gamma_2=1-\frac{u_0^2}{2c^2}. \tag{17}$$

The eq. (16) in dimensional form assumes the form

$$\gamma_3\left(\omega-ku_0\right)^2=\omega_{pe}^2+k^2V_{F_e}+\gamma_2\frac{H^2k^4V_{F_e}^4}{4\omega_{P_e}^2}. \tag{18}$$

Equation (18) gives the relativistic quantum plasma wave dispersion relation. In the absence of relativistic effect i.e. $\gamma_2=\gamma_3=1$, the eq. (18) reduces to quantum electron plasma wave in a non-drifting, $u_0=0$, plasma. For $u_0=0$ and $H=0$, the eq. (18) reduces to the well known dispersion relation of electron plasma waves such that V_{F_e} is replaced by the electron thermal velocity V_e. For a relativistic classical cold plasma the eq. (18) reduces to

$$\omega=ku_0\pm\omega_{P_e}\sqrt{1+3u_0^2/c^2} \tag{19}$$

The group velocity $C_g=d\omega/dk$ is evaluated from eq. (17) as

$$C_g=\frac{k+\left(\gamma_2H^2\,k^3/2\right)}{\sqrt{\gamma_3\left(1+k^2\left(1+\frac{\gamma_2H^2k^2}{4}\right)\right)}}+u_0. \tag{20}$$

From the set of eqs. II, the second harmonic quantities in the lowest order may be obtained from the solutions of lowest order equations after substituting perturbation equation (14), one obtains

$$\left.\begin{aligned} \varphi_2^{(1)} &= -b_2\varphi_1^{(1)^2} \\ n_{e_2}^{(1)} &= 4k^2 b_2 \varphi_1^{(1)^2} \\ u_{e_2}^{(1)} &= (\omega - ku_0)(4b_2k - k^3)\varphi_1^{(1)^2}, \end{aligned}\right\} \tag{21}$$

where

$$b_2 = \frac{\left[2(\omega-ku_0)^2 k^3\gamma_3 + k^5\left(1+\gamma_2\frac{H^2k^2}{4}\right) + k^2(\omega-ku_0)^2\left(k\gamma_3 - 3\gamma_1(\omega-ku_0)\right) - \frac{k^6H^2\gamma_1}{4}(\omega-ku_0)\right]}{\left[8(\omega-ku_0)^2\gamma_3 - 8k^3\left(1+H^2k^2\gamma_2\right) - 2k\right]} \tag{22}$$

in which $\gamma_1 = u_0/c^2$.

From the set of equations III after substituting the perturbation eq. (14), the zeroth harmonic quantities are obtained from the solution of the lowest order equation

$$\left.\begin{aligned} \varphi_0^{(1)} &= b_0\varphi_1^{(1)^2}, \\ n_{eo}^{(1)} &= n_{io}^{(1)} = b_1\varphi_1^{(1)^2}, \\ u_{eo}^{(1)} &= \left[b_1(c_g - u_0) - 2(\omega-ku_0)k^3\right]\varphi_1^{(1)^2}, \\ u_{io}^{(1)} &= b_1(c_g-u_0)\varphi_i^{(1)^2}, \end{aligned}\right\} \tag{23}$$

where

$$b_0 = \frac{k^2\left[2\gamma_3(c_g-u_0)k(\omega-ku_0) + \left\{\gamma_3 + 3\gamma_1(u_0-c_g)\right\}(\omega-ku_0)^2 + \left(1+\frac{\gamma_2H^2k^2}{4}\right)k^2 + \frac{3\gamma_1H^2k^3}{4}(\omega-ku_0)\right]}{\left[(c_g-u_0)^2\gamma_3 - 1\right]}, \tag{24}$$

$$b_1 = \frac{\mu b_0}{\left[(c_g-u_0)^2\gamma_3 - \sigma\right]}. \tag{25}$$

The first harmonic quantities in the second order are obtained from the solutions (15) by replacing $-i\omega$ by $\left(-i\omega - \in c_g\frac{\partial}{\partial\xi}\right) + \left(\in^2\frac{\partial}{\partial\tau}\right)$ and i k by $\left(ik + \in\frac{\partial}{\partial\xi}\right)$ and then picking out ξ terms. Hence, we get

$$\left.\begin{aligned} \varphi_1^{(2)} &= 0 \\ n_{e_1}^{(2)} &= 2ik\frac{\partial\varphi_1^{(1)}}{\partial\xi}, \\ u_{e_1}^{(2)} &= i(\omega + kc_g - 2ku_0)\frac{\partial\varphi_1^{(1)}}{\partial\xi}. \end{aligned}\right\} \tag{26}$$

Again by collecting coefficients of $\in^3$ from both sides of the sets of equation I after substituting eq. (14), we obtain a set of equations for the first harmonic quantities in the third order. In view of the above solutions and after suitable elimination, we get the nonlinear Schrodinger equation (NLSE) as

$$i\frac{\partial\alpha}{\partial z} + P\frac{\partial^2\alpha}{\partial\xi^2} = Q\alpha^2\alpha^*, \tag{27}$$

where $\alpha = \varphi_1^{(1)}$, the group dispersion coefficient

$$P = \frac{1}{2}\frac{dc_g}{dk} = \frac{\left[1 - \gamma_3(c_g-u_0)^2 + \frac{3}{2}\gamma_2H^2k^2\right]}{2\gamma_3(\omega-ku_0)}, \tag{28}$$

and the nonlinear coefficient

$$Q = \frac{\left[\gamma_3(\omega-ku_0)f_1 + k(f_2+f_3)\right]}{2k^2\gamma_3(\omega-ku_0)}, \tag{29}$$

where

$$\left.\begin{aligned} f_1 &= -k\left[(\omega-ku_0)\left(b_1k - 3k^5 + 8b_2k^3\right) + b_1k^2(c_g-u_0)\right] \\ f_2 &= \left(-k^2(\omega-ku_0)\left\{b_1(c_g-u_0) - 2(\omega-ku_0)k^3\right\}\right. \\ &\quad \left. - \gamma_3k^2(\omega-ku_0)^2\left(4b_2k-k^3\right)\left(\gamma_3 + \frac{3u_0^2}{c^2}\right)\right) \backslash \\ &\quad - \frac{9u_0k^4}{2c^2}(\omega-ku_0)^2 - b_1k^3 - 4b_2k^5 \\ f_3 &= \frac{k^4H^2}{8}\left[\gamma_2\left\{b_1\left(k^2+20b_2k^4\right)\right\} - \frac{1}{c^2}\left(2k^2u_0\right)\left\{b_1(c_g-u_0) - 2k^3(\omega-ku_0)\right\}\right. \\ &\quad \left. + \frac{1}{c^2}\left\{72u_0k^3b_2(\omega-ku_0) + k^4(\omega-ku_0)^2\left(1-\frac{2}{c^2}\right)\right\}\right] \end{aligned}\right\} \tag{30}$$

It is an important to note that both the coefficients P and Q depend on the relativistic effects through the terms γ_2, γ_3 and u_0.

4. Modulational Instability

The nonlinear Schrodinger Equation (NSE) describes the amplitude modulation of electron plasma waves with quantum corrections and relativistic effects. Now it is obvious that when the wave is modulationally stable, whereas when the wave becomes modulationally unstable.

5. Concluding Remarks

We have presented one-dimensional quantum hydrodynamic (QHD) model to study relativistic effects on the linear and nonlinear properties of quantum electron plasma waves in a two component electron-ion dense quantum plasma with the effects of ion motion. We have taken weakly relativistic situation. Electrons, due to their lighter mass, gain relativistic speed very easily than ions having heavier mass. Hence, we have considered ion motion as non-relativistic and presented relativistic effects on electron motion through the parameter γ_e. For P = 0, we get $i\frac{\partial\alpha}{\partial\tau} = Q\alpha^2\alpha^*$ and for Q = 0, $i\frac{\partial\alpha}{\partial\tau} + P\frac{\partial^2\alpha}{\partial\xi^2} = 0$, where P and Q denote group dispersion coefficient and nonlinear coefficient respectively.

References

1. M. Marklund, and P. K. Shukla, *Rev. Mod. Phys*., **2006**, 78, 591.
2. T. S. Gill, A. S. Baiuns, and N. S. Saini, *Cand. J. Phys*., **2009**, 87, 861.
3. B. C. Kalita, R. Das. and H. K. Sarmah, *Phys. Plasma*, **2011**, 18, 012304.
4. H. Pakzad, *Astro-phys. Space Sci.*, **2011**, 332, 269.
5. R. Saeed, A. Shah, and M. N. Haq, *Phys. Plasma.*, **2010**, 17, 102301.
6. H. N. Han, S. L. Du and W. S. Duan, *Phys. Plasma*, **2008**, 15, 112104.

7. B. Sahu and Chowdhury, R. Roy, *Phys. Plasma*, **2006**, 13, 072302.
8. T. S. Gill, A. Singh, H. Kaur, N. S. Saini, and P. Bala, *Phys. Lett.*, **2007**, A361, 364.
9. P. K. Shukla and B. Eliasson, *Phys. Rev. Lett*., **2006**, 96, 245001.
10. G. Manfredi, *Fields Inst. Commun.*, **2005**, 46, 263.
11. A. P. Misra and C. Bhowmik, *Phys. Plasma*, **2007**, 14, 012309.
12. S. A. Khan and A. Mushtaq, *Phys. Plasma*, 2007, 14, 083703.
13. A. P. Mishra and Chowdhury, R. Roy, *Phys. Plasma*, **2006**, 13, 072305.
14. B. Ghosh, S. Chandra and S. N. Paul, *Phys. Plasma*, **2011**, 18, 012106.
15. B. Sahu, *Pramana-J. Phys.*, **2011**, 76, 933.
16. T. S. Gill, A. S. Bains and C. Bedi, *J. Phys. Conf. Ser.*, **2010**, 208, 012040.
17. G. Manfredi and F. Haas, *Phys. Rev.*, **2001**, B64, 075316.
18. F. Hass, *Phys. Plasma*, **2005**, 12, 062117.
19. C. L. Gardner and C. Ringhofer, *Phys. Rev.*, **1996**, D53,157.